

The QCD Critical Point : marching towards continuum

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Abstract

We present results of our simulations of QCD with two light dynamical quarks on a $32^3 \times 8$ lattice at a current quark mass tuned to have the Goldstone pion mass of about 230 MeV. Employing the Taylor expansion method we proposed earlier, we estimate the radius of convergence of the series for the baryonic susceptibility by using terms up to eighth order. Together with our earlier results, corresponding to the same physical parameters but on coarser lattices at respectively 1.33 times and twice the lattice cut-off (a), we were able to attempt a march towards the continuum limit.

Keywords: QCD Critical Point, Lattice QCD, Baryonic Susceptibility, Continuum limit.

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1. Introduction

Whether the phase diagram of strongly interacting matter, governed by Quantum Chromo Dynamics (QCD), has a critical point in the temperature (T) and baryon chemical potential μ_B plane, is an exciting question that has attracted many theorists, phenomenologists, and heavy ion experimentalists. A variety of models, which have been successfully tested for hadronic properties in our world, such as an effective chiral Nambu-Jana Lasinio type model, lead to a phase diagram [1] with a critical point in a world with two light quarks and one heavier quark. It is clearly desirable to obtain it from QCD directly, or show a lack of its existence, especially since enormous efforts at RHIC and other accelerators are being devoted to look for it. One usually has to deal with large coupling constants in the world of (low energy) hadronic interactions. Non-perturbative lattice QCD, defined on a discrete space-time lattice, has proved itself to be the most reliable technique for extracting such information from QCD. The hadron spectrum has been computed successfully and predictions of weak decay constants of heavy mesons have been made. Since these fix quark masses and Λ_{QCD} , a completely parameter free application of this approach to finite temperature QCD has yielded a slew of thermodynamics determinations, such as the pressure as a function of temperature. It is therefore natural to ask whether lattice QCD can help us in locating the QCD critical point.

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Due to the well-known fermion doubling problem, one has to make a compromise in choosing the quark type for any computation. We employ the staggered quarks. These have an exact chiral symmetry which provides an order parameter for the entire T - μ_B plane but unfortunately flavour and spin symmetry are broken for them on lattice. The existence of the critical point, on the other hand, is expected to depend crucially on the number of flavours. Although computationally much more expensive, Domain Wall or Overlap Fermions are better in this regard, as they do have the correct symmetries for any lattice spacing at zero temperature and density. Introduction of chemical potential, μ , for these, however, is not straight-forward due to their non-locality. Bloch and Wettig [2] proposed a way to do this. Unfortunately, it turns out [3] that their prescription breaks chiral symmetry. Recently, this problem has been solved [4] and a lattice action with nonzero μ and the same chiral symmetries as continuum QCD has been proposed. It will be interesting to compare its results with those for the staggered quarks.

Finite density simulations needed for locating a critical point suffer from another well known problem. This one is inherited from the continuum theory itself: the fermion sign problem. Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \prod_f \text{Det } M(m_f, \mu_f) , \quad (1)$$

where quarks have been integrated leading to the determinant. The thermal expectation value of an observable O is

$$\langle O \rangle = \frac{\int DU \exp(-S_G) O \prod_f \text{Det } M(m_f, \mu_f)}{\mathcal{Z}} . \quad (2)$$

Numerical simulations or analytical computations can be done if $\text{Det } M > 0$ for any set of $\{U\}$. However, $\text{Det } M$ is a complex number for any $\mu \neq 0$. This famous phase/sign problem is a major stumbling block in extending the lattice techniques to the entire T - μ_B plane. Several approaches have been proposed in the past decade to deal with it. Let me provide a partial list: 1) Two parameter Re-weighting [5], 2) Imaginary Chemical Potential [6], 3) Taylor Expansion [7], 4) Canonical Ensemble method [8], and 5) Complex Langevin approach [9]. We employ the Taylor expansion approach [7] to obtain the results discussed in the next section.

2. Lattice Results

Our earlier results were obtained by simulating full QCD with two flavours of staggered fermions of mass $m/T_c = 0.1$ on $N_t \times N_s^3$ lattices, with $N_t = 4$ and $N_s = 8, 10, 12, 16, 24$ [10] and a finer $N_t = 6$ with $N_s = 12, 18, 24$ [11]. From the work of the MILC collaboration [12], we know that our lattices correspond to $m_\pi/m_\rho = 0.31 \pm 0.01$, leading to a Goldstone pion of 230 MeV. In order to approach the continuum limit of $a \rightarrow 0$, we proceeded to work on an even finer lattice 8×32^3 , keeping $m/T_c = 0.1$ fixed. As before, the peak of the Polyakov loop susceptibility was used to define the critical coupling β_c and plaquette determinations on symmetric lattices were used to tune the temperature such that simulation range covered $0.90 \leq T/T_c \leq 2.01$. Our determination of β_c is in excellent agreement with that of Gottlieb et al. [13]. Typically 100-200 independent configurations, separated by many autocorrelation lengths were used to make measurements of physical quantities.

From canonical definitions of number densities n_i and susceptibilities χ_{n_u, n_d} , the QCD pressure P can be seen to have the following expansion in μ :

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu, T)}{T^4} - \frac{P(0, T)}{T^4} = \sum_{n_u, n_d} \chi_{n_u, n_d} \frac{1}{n_u!} \left(\frac{\mu_u}{T} \right)^{n_u} \frac{1}{n_d!} \left(\frac{\mu_d}{T} \right)^{n_d} \quad (3)$$

where the indices n_u and n_d denote the number of derivatives of the partition function with respect to the corresponding chemical potentials. We set $\mu_u = \mu_d = \mu_B/3$ and $m_u = m_d$ in the expressions and construct a series for baryonic susceptibility from this expansion [10]. Its radius of convergence is what we look for.

Successive estimates for the radius of convergence were obtained by using the ratio method [$r_{n+1/n+3} = \sqrt{n(n+1)\chi_B^{(n+1)}/\chi_B^{(n+3)}T^2}$] and the n^{th} root method [$r_{2/n} = (n!\chi_B^{(2)}/\chi_B^{(n+2)}T^n)^{1/n}$]. We used terms up to 8th order in μ for doing so. A key point to note is that all coefficients of the series must be positive for the critical point to be at real μ , and thus physical. We thus first look for this condition to be satisfied and then look for agreement between the two definitions above as well as their n -independence to locate the critical point. The detailed expressions for all the terms can be found in [10] where the method to evaluate them is also explained. We use stochastic estimators. For terms up to the 8th order one needs 20 inversions of M on ~ 1000 vectors for a single measurement on a given gauge configuration. Work is in progress to double these. Our earlier determination of the critical point on $N_t = 6$ resulted from the constancy for both the ratios defined above at $T/T_c = 0.94$, with all the susceptibilities being positive, leading [11] to the coordinates of the endpoint (E) –the critical point– to be $T^E/T_c = 0.94 \pm 0.01$, and $\mu_B^E/T^E = 1.8 \pm 0.1$ for that lattice.

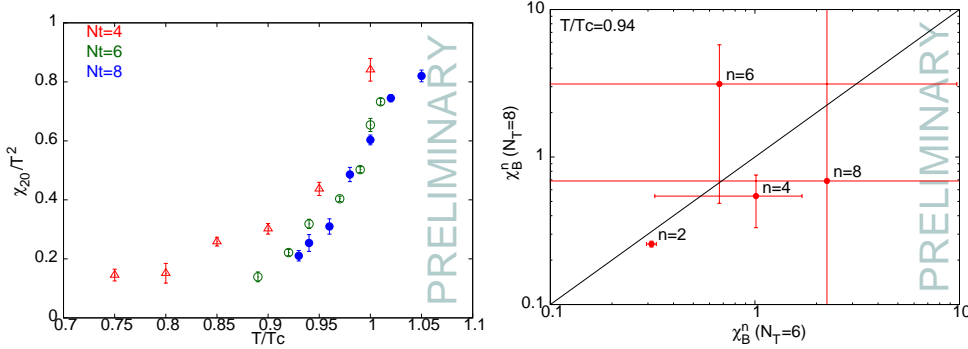


Figure 1: Comparison of baryon number susceptibility on $N_t = 8$ with our earlier results on $N_t = 6$ and 4 (Left panel). Comparison of the expansion coefficients on $N_t = 6$ and 8 lattices. The diagonal solid line displays the ideal case with no cut-off effects. (Right).

The left panel of Fig. 1 shows a comparison of our new results for the $N_t = 8$ lattice for baryon number susceptibility with those for $N_t = 6$ and 4. The encouraging agreement between $N_t = 8$ and 6 suggests that the dimensionless ratios which we employ in all our critical point determinations possess only a mild cut-off dependence. The right panel extends this comparison to all the susceptibilities we determined for the two lattices with $N_t = 8$ and 6. The diagonal line indicates the trajectory of their expected locations in the ideal case of no lattice cut-off effects. This is seen to be so within errors, which still need to be reduced further.

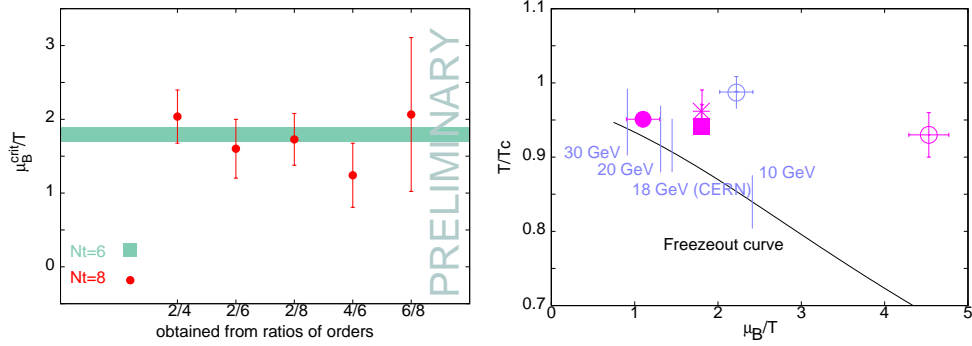


Figure 2: The radius of convergence estimates for $N_t = 8$ (data) along with the $N_t = 6$ critical chemical potential (band) at the critical temperature of $T_E/T_c = 0.94$ (Left panel). QCD phase diagram with all known lattice determinations for critical point (Right). Our new $N_t = 8$ results, presented in this talk, are denoted by the asterisk. $N_t = 4$ [5, 10] and 6 [11] results are shown by the circles and square respectively.

Fig. 2 in its left panel displays our new estimates for the $N_t = 8$, obtained by using the two methods mentioned above and the coefficients in Fig. 1. The solid band indicates our critical chemical potential estimate on $N_t = 6$ and the temperature chosen is the same as the corresponding critical temperature T_E . We see such behaviour in a small band near this temperature, leading to a larger error band on T_E , as exhibited in the QCD phase diagram in the right panel along with our old results for $N_t = 6$ [11] and 4 [10], and those from Budapest-Wuppertal group both [5] of which use $N_t = 4$.

3. Summary

The elusive QCD phase diagram in T - μ_B plane has begun to emerge using first principles lattice approach. Our lattice results for $N_t = 8$ are in very good agreement with those for $N_t = 6$, suggesting the continuum limit to be in sight and the critical point estimate to be robust.

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